

The Gravitational Landscape of the Solar System

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The Sun's gravitational influence is of course much greater than that of any of the planets. Just how much greater can be dramatically illustrated by plotting their combined gravitational potential on the same graph.

The gravitational potential energy of two point masses (or spherically symmetric mass distributions), M and m , is given by

$$U_G(r) = -\frac{GmM}{r},$$

where G is the universal gravitational constant and r is the distance between the centers of the two masses. Thus, every planet or star of mass M creates in the space around it a gravitational potential (potential energy per unit mass)

$$V_G(r) = -\frac{GM}{r},$$

often called a gravitational potential well. This is seldom graphically represented in textbooks. Neither Serway,¹ Giancoli,² nor Hewitt³ shows a plot of this function. Halliday, Resnick, and Walker⁴ do, but only very briefly.

Another mass experiences a force proportional to the negative of the gradient of this potential; thus, a spacecraft would experience a force "downhill" on the potential graph proportional to the steepness of the "hill." A gravitational potential well has the shape of a funnel; understanding orbital motion in terms of motion on the side of a deep, steep, frictionless funnel can give students a way of putting the energetic and

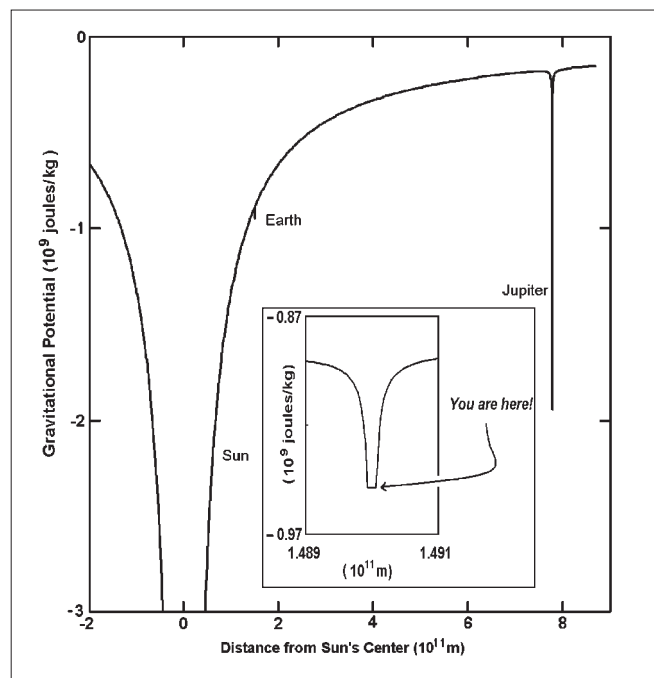


Fig. 1. The gravitational potential of the Sun, Earth, and Jupiter (lying on a single straight line) plotted together on the same scale. The origin is at the center of the Sun. The potential approaches zero as the distance from the Sun approaches infinity. The tiny downward spike at about 1.5×10^{11} m from the center of the Sun is the Earth's gravitational potential well. The downward extent of the spikes for both Earth and Jupiter correspond to the depth of their respective potential wells down to the surfaces of the planets. Note how huge Jupiter's is compared to Earth's, which is why Jupiter has such a capacity for capturing comets. Also note how huge the Sun's well is compared to either planet's. On this scale, the depth of the potential down to the Sun's surface would be over 60 times the vertical extent of the graph! Inset: Enlarged view of the potential in the immediate vicinity of Earth.

dynamic considerations involved into a more familiar context.

If there are multiple astronomical bodies, such as the Sun and its planets, what is the shape of the gravitational potential in which a spacecraft must navigate? Plotting such a potential on a single graph with a planet's gravitational well superimposed on the Sun's dramatically illustrates the enormity of the Sun's gravity as compared with that of the planet.

Assuming a universe containing only the Sun and two planets (Earth and Jupiter) lying along a straight line and setting the origin at the center of the Sun and the potential equal to zero infinitely far away, the gravitational potential along the line connecting the centers of the three bodies is

$$V_G(r) = -\frac{GM_{\text{Sun}}}{|r|} - \frac{GM_{\text{Earth}}}{|R_E - r|} - \frac{GM_{\text{Jupiter}}}{|R_J - r|},$$

where R_E and R_J are the radii of the orbits of the Earth and Jupiter, respectively. This is plotted in Fig. 1.

The pilot of a spacecraft exploring this universe would find it possible to go pretty much anywhere as if on a (nearly) "level playing field" except in the vicinity of the solar system. There the ship would fall in toward the Sun as if sliding down the side of a funnel. Near the bottom of the funnel is the Sun, into which the ship would crash if not careful.

In the case of falling radially inward toward the Sun, to escape would require reversing course and running one's rocket motors sufficiently to reach escape velocity away from the Sun.

Alternatively, the pilot could give the ship a velocity tangent to a circle (or an ellipse) around the Sun so as to end up inside the funnel but circling around on the side of it, never falling any further in. We here on "spaceship Earth" find ourselves in exactly this latter situation.

The inset in Fig. 1 shows an enlargement of the Earth's gravity well. The bottom of the well shown represents the surface of the planet. This is where most of us live our entire lives. All satellites of Earth orbit in circles or ellipses within the funnel that represents the planet's gravity well.

From our perspective on Earth's surface, we seem to be at the bottom of a quite deep and steep well, but in the larger context of the solar system, as seen on the main graph, we are just a mote orbiting around on the

side of a vastly larger funnel. The only thing keeping us from falling into the Sun is our velocity tangent to our circular path. Any spacecraft launched from Earth has approximately this same tangential velocity and therefore is also immune from falling into the Sun. Unfortunately, this also applies to any waste that one might want to get rid of by sending it into the Sun.

References

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3. P.G. Hewitt, *Conceptual Physics: The High School Physics Program*, 2nd ed. (Addison-Wesley, Menlo Park, CA, 1992).
4. D. Halliday, R. Resnick, and J. Walker, *Fundamentals of Physics*, 5th ed. (Wiley, New York, 1997), p. 336.

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